

CONVEX-CONCAVE LANDSLOPES: A GEOMETRICAL STUDY^{1, 2}

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ABSTRACT

The geometry of convex-concave landslopes is described by empirical equations, so that shape, scale, and form elements may be compared numerically. Gradient-length relations determine intervals over which particular form elements exist, and over which particular equations apply. Three segments of slopes are defined, an upper convex element, a middle straight element, and a lower concave element, where the change in gradient with length is respectively positive, zero, and negative. The use of gradient data allows accurate definition of form elements and the intervals over which they exist. Use of profile data alone results in incorrect determination of form elements and inaccurate numerical description.

The slopes studied occur in southwestern Ohio, have straight contours, and are developed on kames, end moraines, and shale hills. Numerical constants obtained from profile and gradient relations accurately describe each slope element. A middle straight element is demonstrated to exist, but is absent in 30 per cent of the slopes studied. Relationships among numerical constants and parent materials indicate highly similar geometric forms, apparently different only in scale. Persistence of equivalent form elements from long to short slopes implies that length of runoff is not the determining factor in shaping convex-concave profiles, and that the presence of the convex element does not depend on dominance of processes of creep over those of runoff.

INTRODUCTION

The Problem

The constituents of landscape are slopes; "If the development of individual slopes is understood, the development of a landscape can be synthesized" (Scheidegger, 1961, p. 1). However, landslopes are little understood, though they have long been studied. One of the most basic kinds of landslope study is geometric. Before attempting sophisticated explanations of the development of slopes, it is axiomatic that we must accurately describe their geometry, for the acceptability of any theory must rest on its ability to account for the geometry of real slopes.

The present investigation is an attempt to describe the geometric elements, forms, and scale of individual slopes by empirical equations. The slopes studied are those with prominent convex-concave elements and smooth regular profiles, commonly referred to as graded or equilibrium slopes. The concepts of grade and equilibrium have long been applied to slopes, by Davis (1899) and especially by Gilbert (1880) in the past, and more recently by Strahler (1950, p. 676) and others.

Only those slopes which satisfied certain simplifying conditions were selected for study: only slopes possessing essentially straight and parallel contour lines, and only those assumed to be in a steady state, or state of dynamic equilibrium. The steady state is manifested by unaccelerated erosion with no down-cutting by adjacent streams. The data were obtained from measurements on natural landslopes in the Dayton, Ohio, region.

Acknowledgments

The investigation was aided financially by the Ford Foundation. Professor Robert Voelker of the Engineering Department contributed to the study. The following students at Antioch College were field and office assistants: MacDougal

¹Summary presented at the Nov. 1964, meeting of the Geological Society of America in Miami, Florida.

²Manuscript received April 17, 1965.

Palmer, Paul Biegelsen, David Goheen, Jeffrey Kash, Barbara Waite, David Thompson, and Lee Meyer. Professors R. P. Sharp and Roger Hooke of the Division of Geological Sciences, California Institute of Technology, and Dr. Marie Morisawa, Department of Earth Sciences, Antioch College, read the manuscript and offered helpful suggestions.

Regional Setting

The slopes studied are in southwestern Ohio (fig. 1) in Greene, Montgomery, Clark, and Warren counties. The study area encompasses 1000 square miles which includes parts of two major drainage basins, the Miami River basin and the Little Miami River basin. This area is part of the Till Plains section of the Central Lowland physiographic province and is characterized by an irregular array of landforms, which owe their origin principally to Pleistocene glaciation. End moraines, kames, bedrock escarpments, and bedrock gorges cut by meltwater con-



FIGURE 1. Location of measurements in southwestern Ohio.

stitute the local relief. The pre-glacial topography has been buried or modified extensively by glaciation. Most of the soils are developed in material of glacial origin: moraines and glaciofluvial materials consisting of clay, sand, and gravel. Flat-lying sedimentary rocks, limestone, dolomite, and shale of Silurian and Ordovician age, comprise the bedrock.

The climate, regarded by most investigators as an important factor in slope processes, is humid continental. The average annual precipitation is 37 inches, which is well distributed throughout the year. The average annual temperature is 53° F; January, the coldest month, has an average temperature of 30° F; July the hottest month, has an average temperature of 75° F. The extreme range of individual temperatures over an 18-year period in Greene County is -23° F to 100° F.

Field Data

The field data, collected during May to November, 1961, consist chiefly of measurements of consecutive increments of vertical distance (fall) over ten-foot intervals of horizontal distance. The measurements, taken downslope and per-

pendicular to the contour lines, were made by using a specially constructed straight-edge equipped with a level bubble and an adjustable steel tape for measuring vertical distance to the nearest 10th of a foot. In all, 72 slopes were measured.

The selection of slopes was based on the conditions previously stated. Some additional considerations were that as wide a range as possible of length, height, gradient, and parent material should be represented; and in so far as possible, the parent material should be uniform over the entire slope length. It was preferable to start measurements at the actual crest, but measurements were made also from various points on the slope.

In processing the data, each increment of fall (ΔH) was divided by 10 giving an average gradient over each 10-foot interval. Each average gradient was treated as an approximate value of the instantaneous gradient at the mid-point of the 10-foot interval. The instantaneous gradients were then plotted as a function of horizontal distance from the crest on arithmetic and "log-log" paper. The increments of fall were also summed to yield fall (H-values) over length. The horizontal length L and fall H were plotted to produce the profile. The profile and gradient data for each slope were expressed by empirical equations. The resulting constants were tabulated; these yielded numerical descriptions of the geometry of each slope.

FORM ELEMENTS

There is a commonly recognized tripartite classification of landslopes which have a soil or lithosol: an upper convex element, a middle straight element, and a

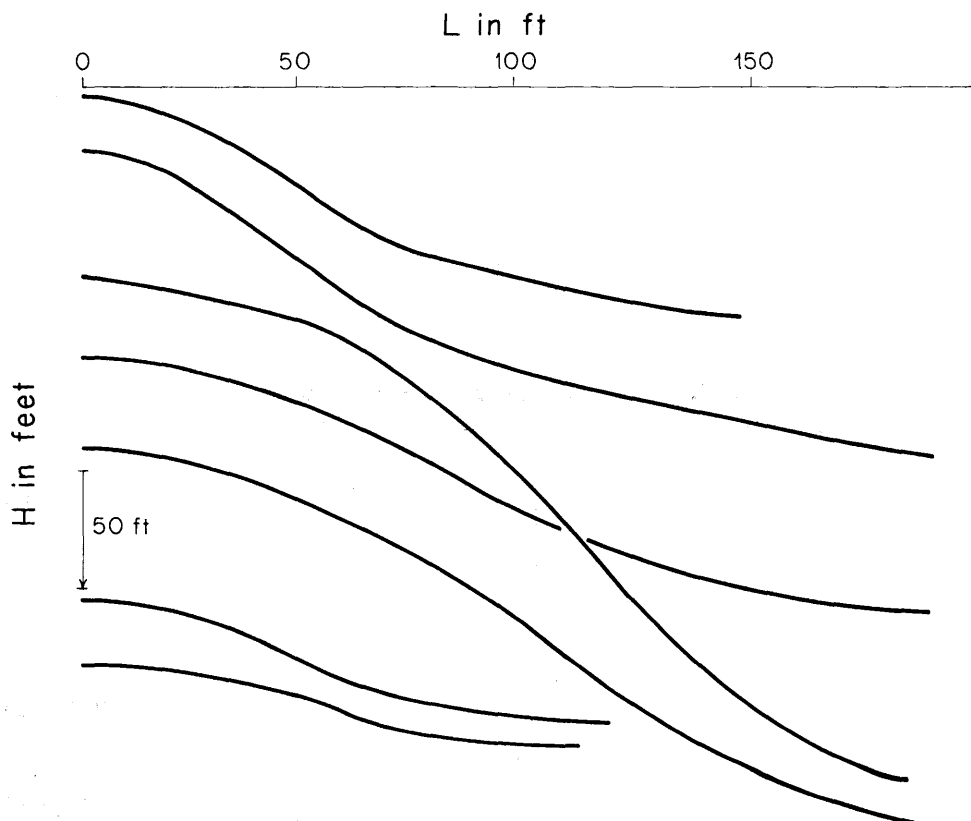


FIGURE 2. Sample profiles. Note smoothness and similar form, but contrasting size, gradient, and shape.

lower concave element. Sparks (1960, p. 63) goes so far as to say "There is a general consensus of opinion that slopes usually consist of a convex upper part and a concave lower part with, very often, a straight slope in between." Gilbert (1909) discussed concave and convex slopes, and Wood (1942) proposed four general basic slope elements which have been applied widely. King (1953), following Wood, lists the slope elements as: waxing slope (convex crest), freeface, talus or debris slope, and waning slope (concave). Twidale (1960) has suggested the terms: upper slope, bluff, debris slope, and planate slope. The slopes of the present study, as judged by their plotted profiles (see examples in figure 2), are smooth regular curves. Their upper portions are prominently convex, their lower portions concave, but the character or existence of a middle element cannot in general be determined from the profiles. The forms of all the profiles, regardless of varying height, length, steepness, or underlying material, are similar and they seem to conform to the common convex-concave type.

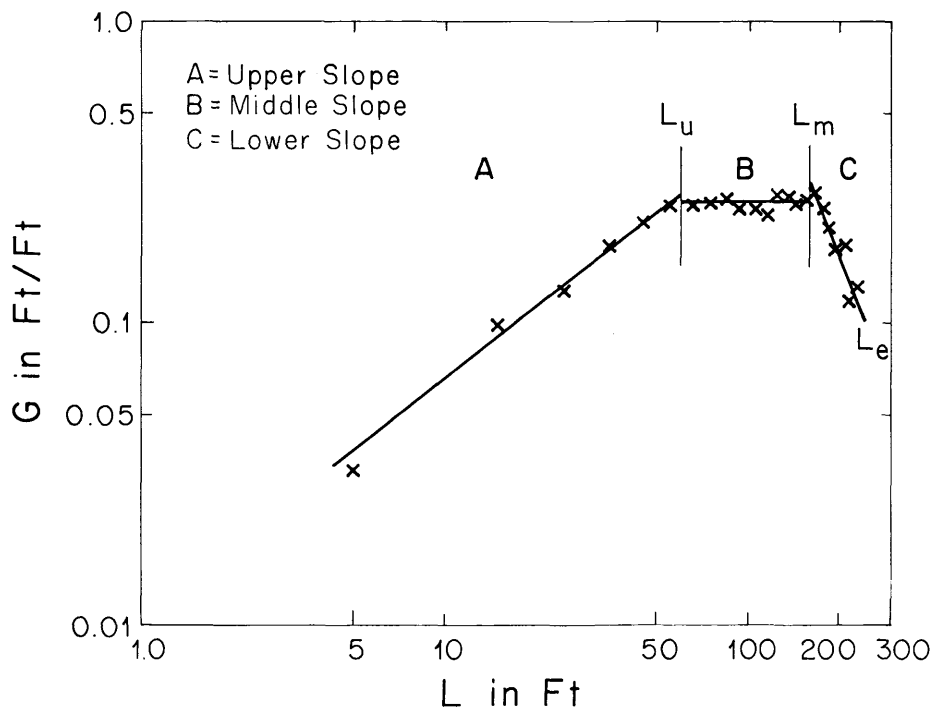


FIGURE 3. Form elements and their graphical determination as a function of length. G is gradient; L is horizontal length from crest. Curve fitted by eye.

In order to gain a more precise analysis of slope profiles and their elements, the nature of the change in gradient over slope length must be considered. If the commonly observed slope or form elements (convex, straight, concave) represent accurate observations, then the gradient should increase over the convex portion, be constant over the straight portion, and decrease over the concave portion of the slope.

To see how the gradient actually does change over length, the gradient-length graphs (either arithmetic or log-log) were plotted for all the slopes. They present a general pattern which is illustrated by the example in figure 3. In figure 3, there are three well-defined form elements, marked by endpoints L_u , L_m , and L_e . The form or slope elements are determined where changes in gradient with respect

to length are positive, zero, and negative. These changes correspond to and define an upper convex slope, where gradient increases with length; a middle straight slope of constant gradient; and a lower concave slope of decreasing gradient. The relations discussed and illustrated by figure 3 apply in general to all slopes studied, except that not all have a discernible middle element.

Numerical values may be easily obtained for the endpoints of the slope elements and each element may then be described by equations derivable from the gradient or from the profile data. As shown by the example in figure 3, straight lines may be fitted to the set of data (L, G) for each slope element. These correspond to power functions of the general form:

$$G = k n L^{n-1} \tag{1}$$

where G is gradient, L is length, and k and n are empirical constants for a given element. Since $dH = G dL$, the solution gives the profile equation:

$$H = k L^n + H_0 \tag{2}$$

where H is fall, L is length, and k, n, and H_0 are constants for the given slope element. As can be seen from the gradient equation (1) and noted by Hack (1960b) and others, the exponent n is a numerical index which defines the shape

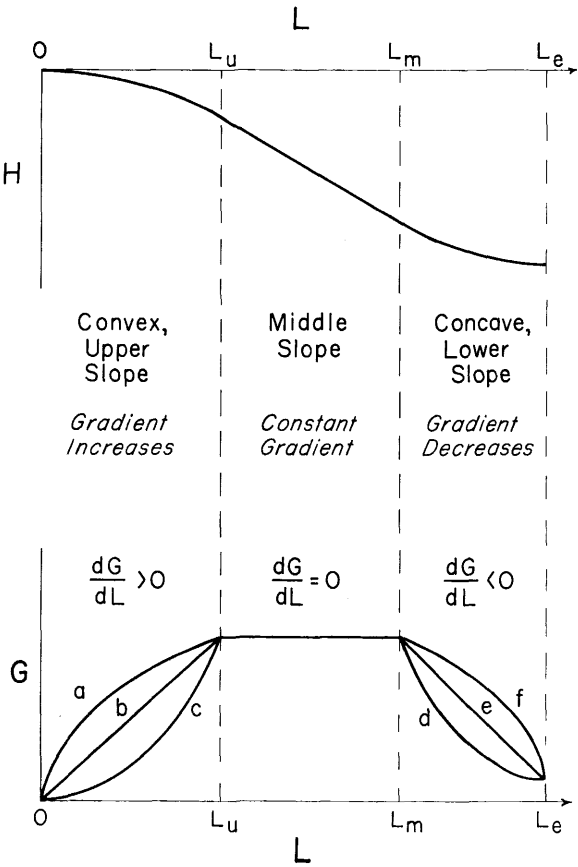


FIGURE 4. Diagram of characteristics of the three form elements of slopes. L is length; G is gradient; and H is fall. The middle element is not always present. a, b, c and d, e, f represent alternate possibilities.

of the profile element: for if $n > 1$, the profile is convex; if $n = 1$, the profile is straight; and if $n < 1$, and $n \neq 0$, the profile is concave. A diagrammatic representation of the slope elements and their characteristics is given in figure 4.

Early in the study, plotting of the profile data on logarithmic paper demonstrated that this data could be fitted approximately with a series of straight-line segments which correspond to power functions, an approach followed by Hack (1960b). Although this approach is useful, there are certain difficulties in its use for analysis of slopes. To illustrate these difficulties, a sample profile, which is plotted in logarithmic form, is shown in figure 5. Lines a, b, and c were fitted to the profile data, and determine inflection points, I_1 and I_2 , which might be taken as marking the endpoints of slope elements. However, they do not cor-

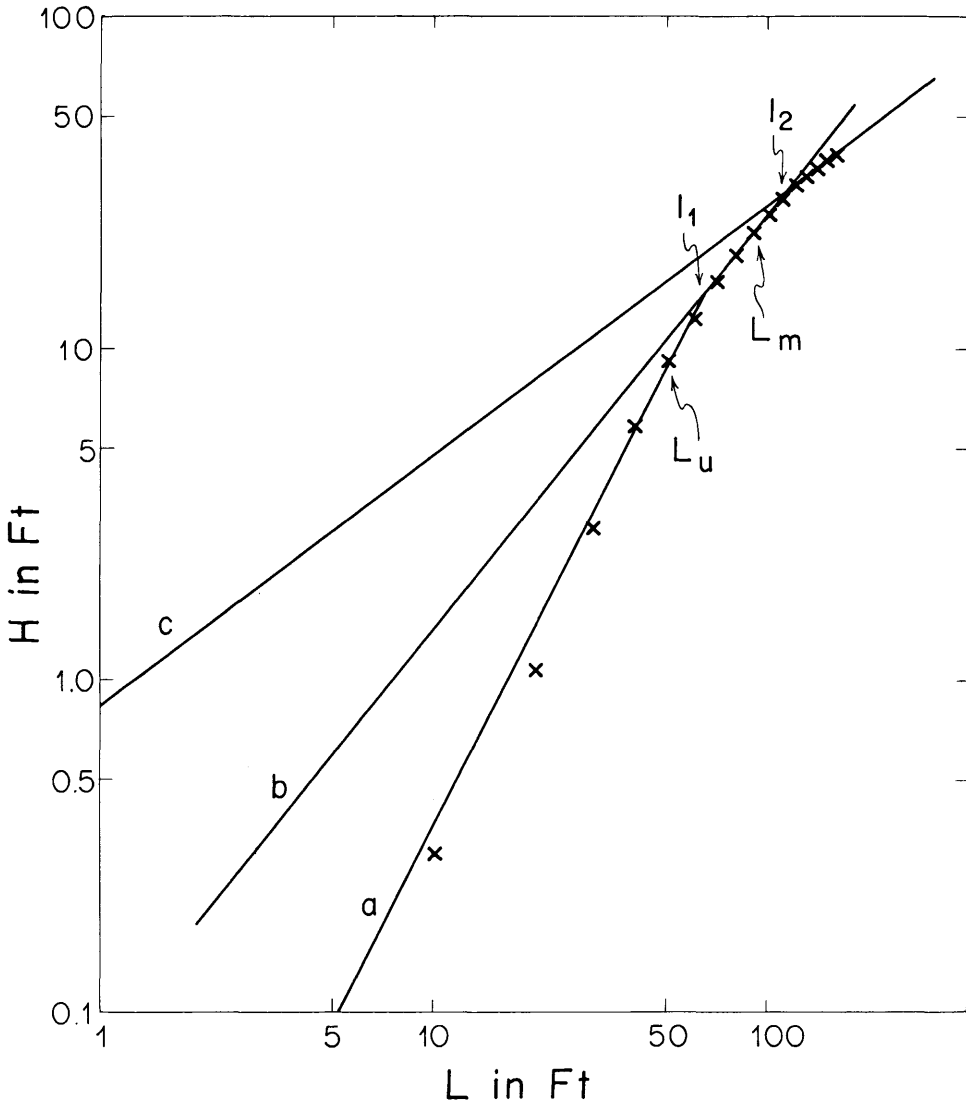


FIGURE 5. Plot of a slope profile to which lines a, b, and c have been fitted. L_u and L_m mark the ends of the upper and middle slopes and were obtained from a gradient-length graph. I_1 and I_2 are inflection points.

respond to the endpoints, L_u and L_m , derived from a gradient-length graph similar to figure 3. If only the plot of the profile were used, erroneous conclusions could be drawn. The lengths of the slope intervals, upper, middle, and lower, would be incorrect as shown by the example in figure 5. While the existence of a middle slope is shown in this example, it is often masked, and in any case the characteristic of constant gradient is not apparent. Another difficulty is that the fitting of curves would not be accurate, even for the upper slope, and numerical values for inflection points and constants for derived equations would have little meaning except possibly for those for the upper slope. Also there is often a problem of how many lines to fit to the profile plot. In addition, it should be noted that the loci for the middle and lower elements are not straight but curved lines, as shown particularly by the lower element in figure 5. The case illustrated by figure 5 is a general one for the slopes studied, and does not illustrate the maximum errors that occur in using only profile data.

In summary, the importance of the gradient-length relation is that it allows a more accurate determination of form elements, because the logarithmic plot of the profile is relatively insensitive to changes in shape from convex to straight and from straight to concave.

UPPER SLOPES

Profile and Gradient Relations

Although the equation for the profile of an upper slope can be derived from the gradient data, the preferred procedure is to obtain it from the profile, after first establishing the upper slope interval from the gradients. In figure 6, an example is shown of plotting the profile on log-log scales; the endpoint L_u was taken from the gradient-length graph. Equivalent results to those shown were obtained for all the slopes measured. The straight line is represented by $\log H = n \log L + \log k$, the logarithmic form of a power function. The form of the power function is the same as that deduced from the gradient equation, where $H_0 = 0$.

$$H = k L^n, 0 \leq L \leq L_u, n > 1 \quad (3)$$

where symbols are as before and $0 \leq L \leq L_u$ is the interval over which the relation is valid. In order to determine the specific equation for a given slope, the coefficient k and exponent n can be easily obtained graphically, or they may be calculated. The lines representing the equation could be accurately fitted by eye, because there was little deviation of plotted values of L and H from a straight line. To visually illustrate the lack of scatter, a profile is compared in figure 7 with that predicted from the equation. Such close agreement is characteristic and is a manifestation of the smoothness and regularity of the actual profiles.

Equation (1), relating gradient and length for the upper element, may be obtained either from the observed gradients or from differentiating the profile equation (3):

$$dH/dL = knL^{n-1} = G, 0 \leq L \leq L_u, n > 1 \quad (4)$$

A useful relation for n of the upper slopes, and one which defines n geometrically, is:

$$n = \frac{dH/dL}{H/L} = \frac{G}{\bar{G}_P} \quad (5)$$

Here, n is equal to the instantaneous gradient, G , at any point P on the upper element divided by the average gradient, \bar{G}_P , from the origin to the point P . In order to determine n , any G on the upper slope may be used along with corresponding values of L and H . The profile and gradient equations for the upper slope may then be determined in this alternate way. Equation (5) also may be used

to solve for either the maximum or average gradient, for the maximum gradient (G_u) will be equal to the gradient at the endpoint (L_u) of the upper slope, i.e., $G_u = n \bar{G}$.

As previously mentioned, n is an index which describes the shape of the profile

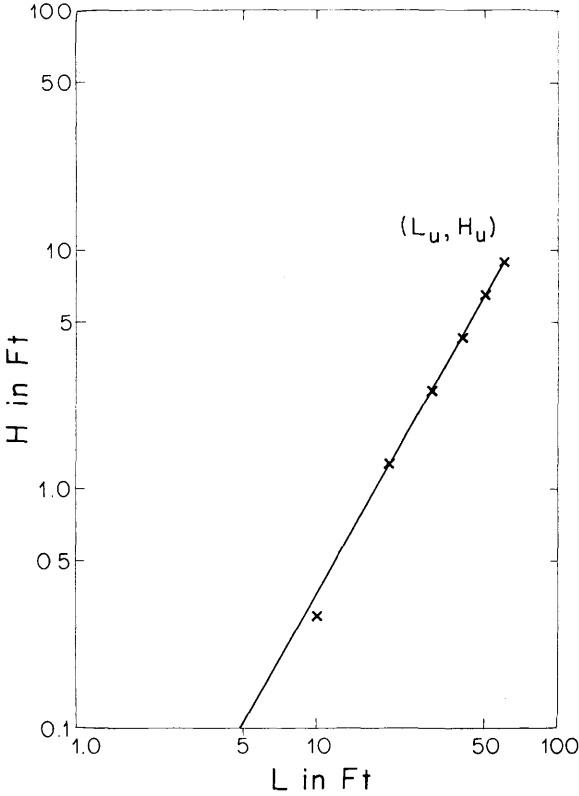


FIGURE 6. Measured (L, H) plotted on logarithmic scales. The straight line, representing the plot of a power function, was fitted to the data by eye.

element (convex, straight, or concave); in addition, n may be used to define numerically the shape of an individual convex slope or its degree of convexity (Hack, 1960b). If a family of convex slopes of constant average gradient is considered, n is a measure of their departure from a straight line. Slopes of the

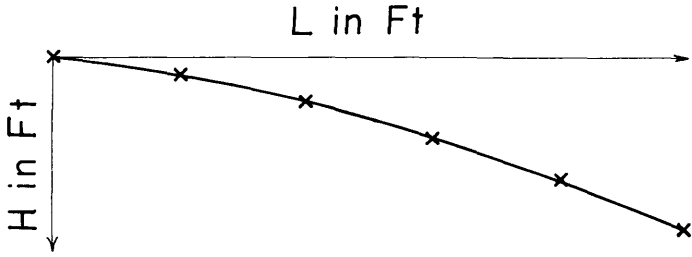


FIGURE 7. Comparison of the observed and calculated profile of an upper slope. The line represents measured values; the crosses (L, H) values derived from the power function. Length of this upper slope is 50 feet.

same average gradient but higher *n* will have greater maximum gradients regardless of size (height or length). Because *n* defines the general profile shape and is also a factor in the shape of the upper slope, *n* is proposed as a *form-index*; *n* is a dimensionless constant and thus is a valid index of slope shape regardless of scale.

Geometric Data for the Upper Slopes

The equation for each upper slope was written from the values of *k*, *n*, and *L_u*; for example, the equation for a given profile, by substitution in equation (3) becomes:

$$H = 0.0090 L^{1.65}, 0 \leq L \leq 80.$$

The equation for the gradient from equation (4) is:

$$G = 0.0149 L^{0.65}, 0 \leq L \leq 80.$$

The maximum gradient, *G_u*, i.e., *G* at *L_u*, is:

$$G_u = n H_u/L_u \text{ or } G_u = n \bar{G};$$

$$G_u = 1.65 (0.144) = 0.238.$$

The ranges of the geometric data are summarized in table 1. They indicate that the equations are applicable to both small and relatively large slopes, and also to gentle and steep slopes. The equations apply to the full range of the observations and to slopes on kames, end moraines, and shale. No data were obtained for upper slopes on limestone because of the prevalence of scarps. The values of the form-index *n* are all greater than one, indicating that the upper slopes are all convex. The range in the coefficient *k* is attributed to a large range in size (i.e., both length and height) of the upper slopes and to the range in *n*.

TABLE 1
Ranges in upper slope data

	Minimum	Maximum
Maximum gradient, <i>G_u</i> (equivalent slope angle)	0.049 ft./ft. 2° 50'	0.52 ft./ft. 27° 30'
Average gradient, <i>H_u/L_u</i> (equivalent slope angle)	0.031 1° 50'	0.34 18° 50'
Height (fall) of upper slope, <i>H_u</i>	1.1 ft.	36.3 ft.
Length of upper slope, <i>L_u</i>	30 ft.	170 ft.
Form-Index, <i>n</i>	1.2	2.4
Coefficient, <i>k</i> (Number of upper slopes measured=35)	0.00035	0.113

As a means of display, *k* is plotted against *n* in figure 8. A strong trend is apparent, such that as *n* becomes larger, *k* becomes smaller, and that for any given *n*, there is a large but restricted range of *k*. A diagram similar to figure 8 was given by Hack (1960b) for an investigation encompassing slopes in a different geographic region. Comparison of the two diagrams shows that the results are remarkably similar, particularly if one considers the different materials, terrains, and climates associated with the two sets of data. The upper slopes of the present study, as well as those reported by Hack, are broadly similar over a large range of variable factors, including different parent rock types, climatic regimes, biotic factors, sizes of slopes, and relief.

Figure 9 portrays the relation between maximum gradient, average gradient,

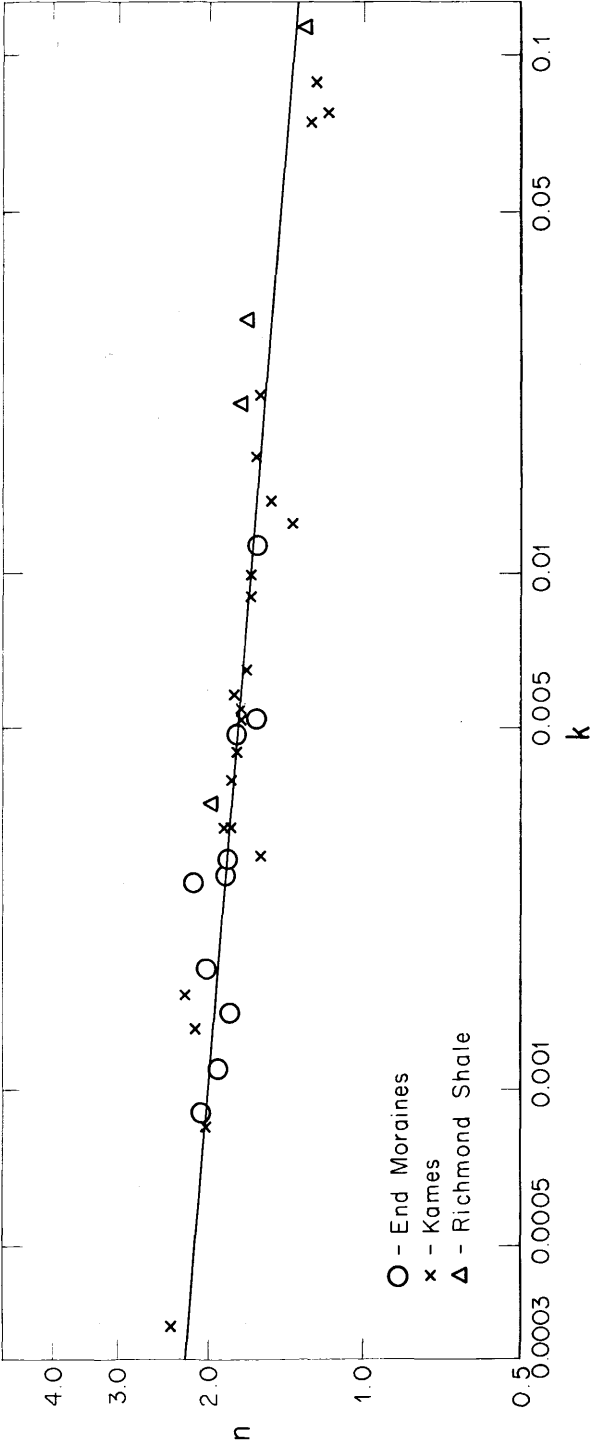


FIGURE 8. Coefficients, k , and exponents, n , of upper slopes. Trend-line fitted by eye.

and n . The distribution of the data suggests that n is independent or at least not strongly dependent on the gradient; and the maximum gradient varies directly with the average gradient. While both maximum and average gradient are variable over a wide range, n is much more constant. Although n ranges from 1.2 to 2.4, most of the values are between 1.5 and 2.0. The mean n for 35 slopes is 1.74, with a standard deviation of 0.28.

Some of the scatter of the n values is attributed to errors. The chief error was in the uncertainty of the location of the origin, which was not determined by measurement but located by eye at the crest. With better procedure, this error could be considerably reduced. The degree of regularity of the slope surfaces and the sensitivity of n to position of the origin was not anticipated. Making corrections where possible for large errors in the location of the origin, the best mean values for the measurements are the following:

21 kames, $\bar{n} = 1.76$, standard deviation 0.22; range 1.4 to 2.2;

10 end moraines, $\bar{n} = 1.91$, range 1.6 to 2.1;

4 shale (Richmond) slopes, $\bar{n} = 1.88$, range 1.7 to 2.1.

All n values for end moraines and shale slopes fall within the range of n for kames. Apparently slopes developed on different parent materials have mean n values that are similar and may not be significantly different.

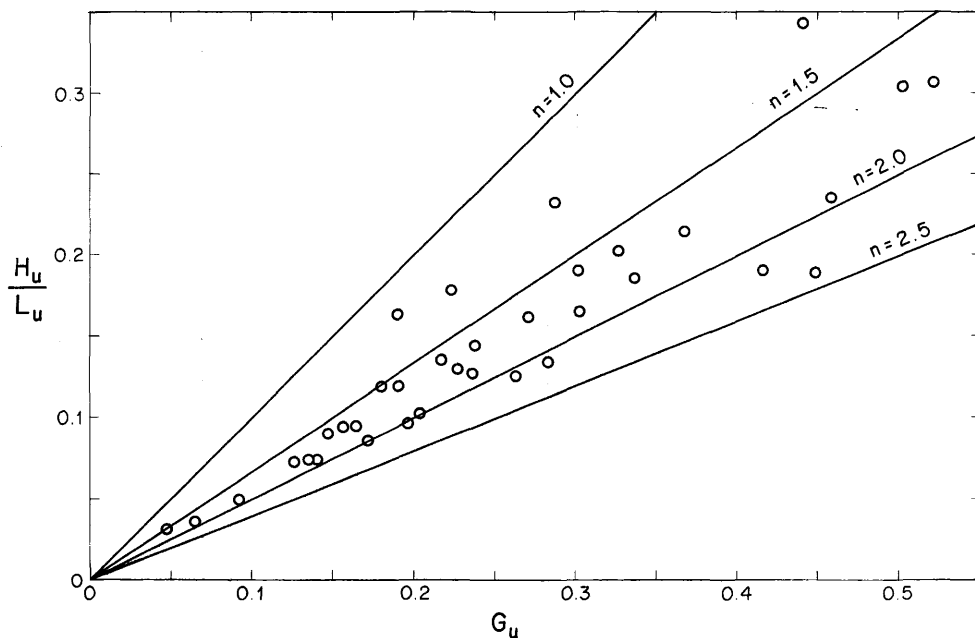


FIGURE 9. Relationships of maximum gradients, average gradients, and n -values for upper slopes.

Scatter diagrams (fig. 10) demonstrate a lack of correlation of n with length and height of the upper slope. Similar results were shown among other geometric parameters. The maximum and average gradients were plotted against height, length, and parent material. The scatter diagrams, not shown because of negative results, were similar in pattern to those of figure 10. In summary, neither length, height, steepness, nor parent material acts as a dominant or controlling factor in determining either n or the maximum and average gradients.

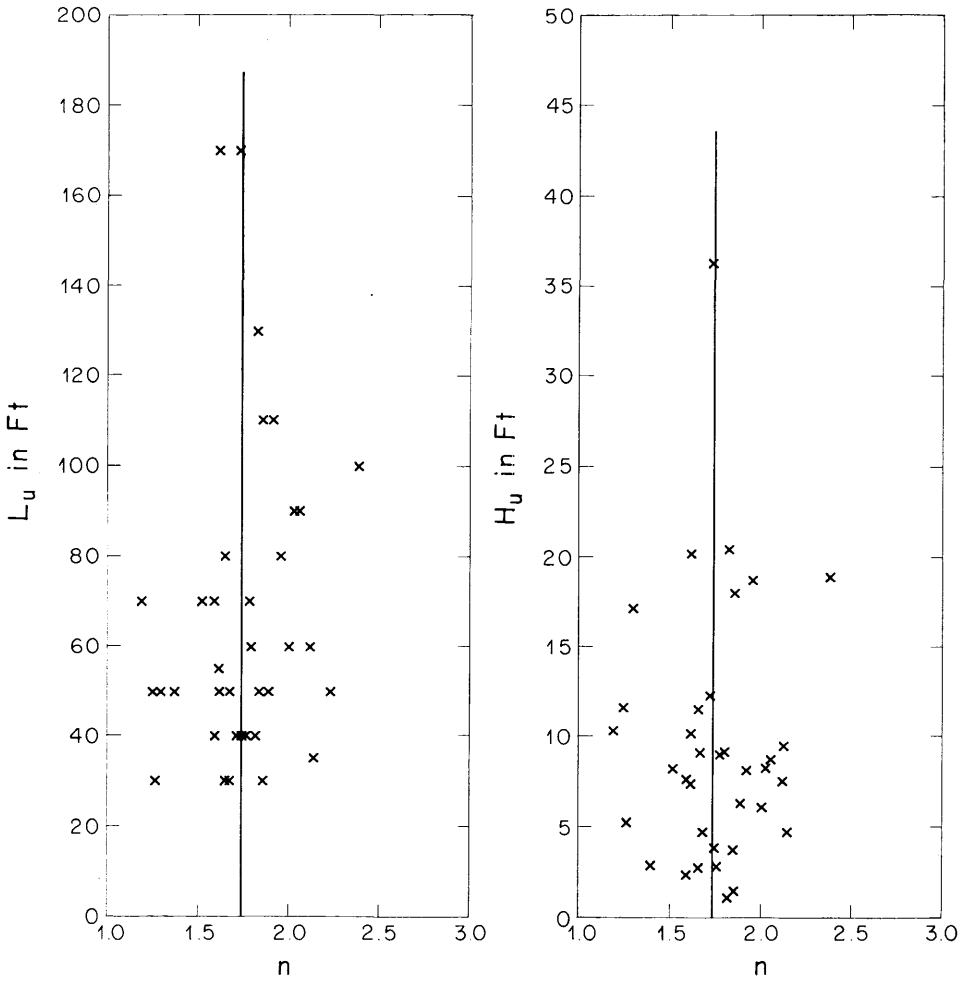


FIGURE 10. Length of upper slope (L_u), and form-index n . Height of upper slope (H_u), and form-index n .

MIDDLE SLOPES

Profile and Gradient Relations

In order to portray the middle slopes by a function of the form $H = k L^n$, a new origin was selected at L_u, H_u . Paired values (L', H'), obtained by subtracting the constants L_u and H_u from L and H , were plotted for each middle slope, as shown by an example in figure 11. The example illustrates how closely the data approximate a straight line and an equation of the form $H' = k_m (L')^n$. This equation, when transformed to the original coordinates, becomes:

$$H = H_u + k_m (L - L_u)^n, \quad L_u \leq L \leq L_m. \quad (6)$$

The equation for the gradient is:

$$G = k_m n (L - L_u)^{n-1}, \quad L_u \leq L \leq L_m. \quad (7)$$

If $n_m = 1$, then

$$H = H_u + k_m (L - L_u), \quad L_u \leq L \leq L_m. \quad (8)$$

and

$$G = k_m, \quad L_u \leq L \leq L_m. \quad (9)$$

If the middle element closely approximates a constant gradient slope, then n will be close to one. In order to gain a measure of how closely n approximates one, the values of n were calculated from the profiles of 14 slopes which had well developed middle elements as shown by the gradient-length graphs. The calculated values ranged from 0.98 to 1.03, with a mean value of 1.00. It is concluded that the middle slopes tend to approach closely the theoretical straight slopes of constant gradient.

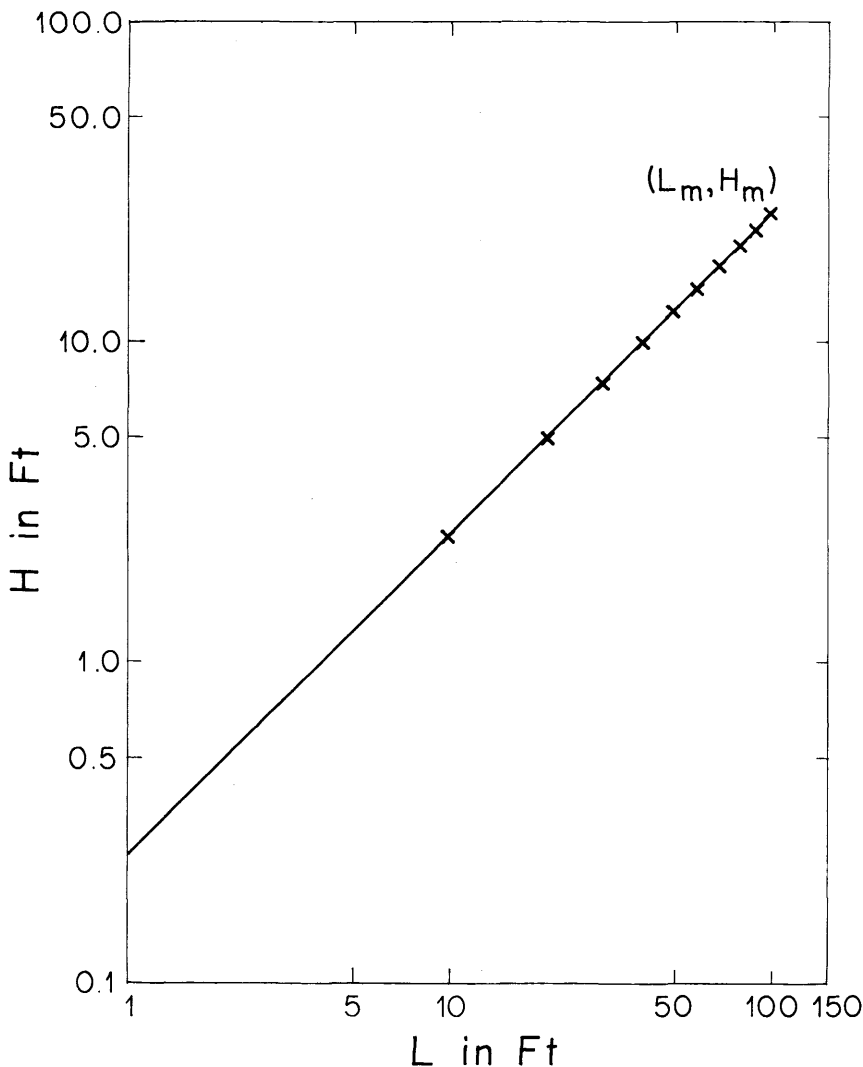


FIGURE 11. Profile of a middle slope transferred to an origin at the endpoint of the upper slope (L_u, H_u).

Geometrical Data for Middle Slopes

The geometrical data are summarized in table 2. L_u , H_u and L_m , H_m are endpoints which define the intervals over which the middle slope exists; i.e., intervals over which, theoretically, dG/dL is equal to zero, dH/dL is constant, the profile is straight, and the form-index n is equal to one. As seen from table 2,

the straight segment is not confined to steep slopes, for the slope angles range from 2 to 28 degrees, nor is it restricted to any narrow range of length or height, nor to any particular parent material. The data do not entirely correspond to those of the upper slopes (table 1): e.g., some elements were not measured or did not yield usable data, and not all the slopes had a discernible middle element. About 50 per cent had well defined middle intervals, such as shown in figure 3; 20 per cent more were regarded as probably having straight elements; 30 per cent either had middle intervals too small to be determined or did not have constant-gradient elements.

A comparison of gradients of individual slopes with their parent materials shows that the ranges of the gradients for any given material are large: kames, 0.05 to 0.40 (17 slopes); end moraines, 0.04 to 0.28 (12 slopes); limestone, 0.22 to 0.50 (9 slopes); shale, 0.21 to 0.54 (9 slopes).

TABLE 2
Ranges in middle slope data

	Minimum	Maximum
Gradient k_m , G _m (equivalent slope angle)	0.042 ft./ft. 2° 20'	0.54 ft./ft. 28° 10'
Form-Index, n^*	0.98	1.03
Height (fall) middle element, H_m-H_u	1.4 ft.	56.8 ft.
Length middle element, L_m-L_u (Number of middle slopes measured=48)	10 ft.	240 ft.

*Calculated for 14 slopes; remaining slopes n assumed to equal 1.00.

LOWER SLOPES

Profile and Gradient Relations

The description of the geometry of the lower slopes was approached by several means, none of which was completely satisfactory. If the profile is plotted on logarithmic paper using the same origin as for the upper slope, it is represented by a curved line. This curved line is approximated by equation (2), $H = k L^n + H_o$, which may be rewritten so that the profile points will approximate a straight line:

$$H' = k \cdot L^n, L_m \leq L \leq L_o, 0 \neq n < 1 \quad (10)$$

where $H' = H - H_o$, H_o may be found by substituting any point (L, H) in equation (2); e.g., $H_o = H_m - k L_m^n$ where L_m , H_m are the profile values at the upper end-point. The plotted points (L, H') in equation (10) approximate a straight line for the slopes studied. A form-index n of less than one was found for each lower slope and means that the lower slopes were concave. Although this procedure demonstrates both the regularity and concavity of the lower slopes, it has disadvantages. First, the fitting of the curve to the data is inherently less exact than with the upper and middle elements, because the greater distance to the origin makes plotting and fitting less accurate when using a logarithmic scale. Second, there is the problem of an origin for each lower slope that depends on the upper and middle slopes. This makes it difficult to compare the constants of the lower slopes, for both k and n depend on the choice of origin.

A method which allows some comparison to be made between numerical values of the constants is based on the observation that the lower elements have a considerable interval, beginning at the end of the middle slope, over which dG/dL is approximately constant. If dG/dL is assumed constant over L , then the gradient and profile can be given by:

$$G = 2 k' \cdot L' \quad (11)$$

$$H' = k' \cdot (L')^2 \quad (12)$$

where k' is a constant for a given lower slope, and L' equals the horizontal distance from a new origin located at $L=L_o$ and $G=O$. The new origin is determined by drawing a mean straight line over the lower slope interval and extending this line to the point where $G=O$, which marks the new origin. The equation of this line is that given by equation (11), and the origin is located where $G=O$ and $L=L_o$. The relations between the coordinate system are:

$$L' = L - L_o$$

$$H' = -(H - H_o)$$

where H_o is the vertical fall from the old to the new origin. Then the equations for the profile and gradient are:

$$H - H_o = -k'(L - L_o)^2, \quad L_m \leq L \leq L_e \tag{13}$$

$$G = 2 k'(L - L_o), \quad L_m \leq L \leq L_e \tag{14}$$

Deviations between observed and calculated values of $H(L)$ for the profiles are small. An illustration is provided by the calculated mean of the absolute deviations for a typical lower slope (0.1 ft) with a maximum deviation (0.3 ft).

The upper endpoint corresponds to the lower endpoint of the middle slope; the lower endpoint, L_e , marks the end of the interval where dH/dL decreases with L in a constant manner. This lower endpoint is marked by either a stream channel at grade, the beginning of another convexity, or a surface of lower gradient. The convexity may mark the advance of a new grade level brought about by a change in local base level, climate, or cultivation, or in some instances, it may represent only an original depositional irregularity, a relict slope produced by glaciation. The surface of low gradient may represent a fourth element of land surface.

Geometric Data for Lower Slopes

A summary of ranges in the data on lower slopes is given in table 3. The lower slopes exist over a wide range of steepness and scale. Differences between

TABLE 3
Ranges in lower slope data

	Minimum	Maximum
Maximum gradient, G (equivalent slope angle)	0.095 ft./ft. 5° 30'	0.61 ft./ft. 31° 30'
Height (fall) of lower element, H_e-H_m	2.8 ft.	41.9 ft.
Length of lower element, L_e-L_m	20 ft.	190 ft.
Coefficient, k' (Number of lower slopes measured = 56)	0.000245	0.00713

slopes are expressed by the coefficient k' , the endpoints L_m and L_e , and the maximum gradient G_m . The coefficient k' is a measure of the change in gradient. The range of k' -values for kames, end moraines, limestone, and shale was similar and about that shown for k' generally (table 3). This suggests that, although parent material may be a contributing factor, it is not a controlling one in determining k' .

SLOPE EVOLUTION AND MORPHOLOGY: DISCUSSION

The data indicate that slopes of small height or length consist of the same form elements (convex, straight, concave) as do relatively large slopes. Similarly,

both gentle and steep slopes, and those developed on different parent materials, possess equivalent form elements (except those developed on limestone where scarps are prevalent). The same geometric parameters describe corresponding form elements of all the slopes studied. The slopes are equivalent in gross morphology, but differ in the numerical values which describe their form elements.

In studying individual form elements, the data in general did not suggest relationships between the geometric variables. For example, values of n for upper slopes showed no relation with size (height or length) or steepness, and both the larger and smaller upper slopes had similar ranges and values for n . Equivalent results were obtained for other variables and for the other form elements. In conclusion, the geometry of small slopes, except for scale, was not demonstrated to be different from that of the large slopes.

Although the geometry of small and large slopes is similar, this does not prove that geometric variables are not factors which, together with other variables, determine the form characteristics of a landslope. This study also failed to show whether parent material is or is not a factor, even though the differences in upper slopes of kames and end moraines (shown by mean n -values of 1.76 and 1.91 respectively) are probably not significant. Certainly neither parent material nor height or length of slope determined slope form in the class of slopes studied. Parent material is regarded as one of several other interrelated factors, such as topography, climate, vegetation, and crustal movement, that acting together will determine slope geometry.

The high degree of similarity, both in gross and in detailed morphology, suggests that the slopes studied, regardless of assumed differing rates of development, size, or nature of their parent materials, tend to adjust to and to maintain certain highly regular, geometric forms. This is regarded as supporting the concept that the types of slopes studied represent a dynamic equilibrium or steady state which is maintained for long periods of slope evolution. The concept of a steady state applied to landforms has been emphasized by Strahler (1950), Hack (1960a), Chorley (1962), and others.

Many of these slopes, because they are developed on accumulations of glacial debris, were formed relatively recently, for the retreat of the last ice from this part of Ohio occurred about 16,000 years ago (Forsyth, 1961). Thus development of the present graded slopes on these features has taken place within less than 16,000 years. Apparently, accidental irregularities are removed quickly during slope evolution, for the slope surfaces are remarkably smooth and regular. Although the same processes act through time in processing rock material and in transporting it, the balance between factors might be expected to change as relief became lowered; for example, with lower relief, weathering might proceed at a different rate. However, the data suggest that such changes, if any, are minor in the family of slopes investigated. It is concluded that the present form of the slopes is one that developed rapidly and, once developed, maintained its characteristics.

It is often stated that landslopes, particularly under arid conditions and after reaching an equilibrium condition, tend to retreat in parallel planes which assume characteristic angles. For example, Schumm (1956, p. 627) concludes that with a degrading stream at their base, badland slopes retreat in parallel planes. In the present work, the gradients range so widely for a given type of parent material that highly characteristic angles are not present, even though parallel retreat may take place.

One of the difficult questions in the general problem of understanding slope processes has been the mode of development and meaning of the typical convex-concave profile most common and best developed in humid regions, but also found in arid regions if proper parent material is present. Presence of contrasting convex and concave elements has been attributed generally to dominance of different

processes in the two domains of the developing slope. The convex part (following Gilbert, 1909) is usually explained by the dominance of processes of soil creep; the concave portion, similar in profile to that of a graded stream, is believed to be caused by action of surface runoff. A general review of these and similar views is given by Sparks (1960, pp. 63-71).

It has been thought that erosion by runoff increases as a function of depth and distance downslope (Horton, 1945), for the rate of discharge of runoff is a direct function of length of slope; and either the velocity or the depth of runoff, or both, would be expected to increase downslope. Thus, the eroding capacity of runoff should be a direct function of slope length. However, Schumm (1956, pp. 631-645) concluded that the development of slopes and their profiles in badlands may not conform to these accepted concepts of runoff action: "The action of runoff and its velocity at any particular point is seemingly unrelated to the increase in discharge at increasing distance from the top of the slope."

If the form elements (convex, concave) depend on the relative dominance of creep or surface runoff, as modern theory dictates, it seems difficult to explain how this effect can be maintained from large to very small slopes. If slopes on a given material, e.g., that of kames, are considered, the distances from crest to the end of the convex slope are found to range from 30 to 170 feet; end moraines, on the other hand, which would represent relatively impermeable material, provide a like range in size. Distances from crest to beginning of the concave slope show an even larger range. In short, the persistence of the same form elements from long to relatively short slopes has two significant implications: the length of runoff cannot be the determining factor in the shaping of convex-concave profiles, and the existence of the convex element is not controlled by the relative dominance of processes of creep over those of surface flow.

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